Identifying Critical Locations in Urban Road Net works Using Network Kernel Density Estimation (Net-KDE): a Spatial Analysis

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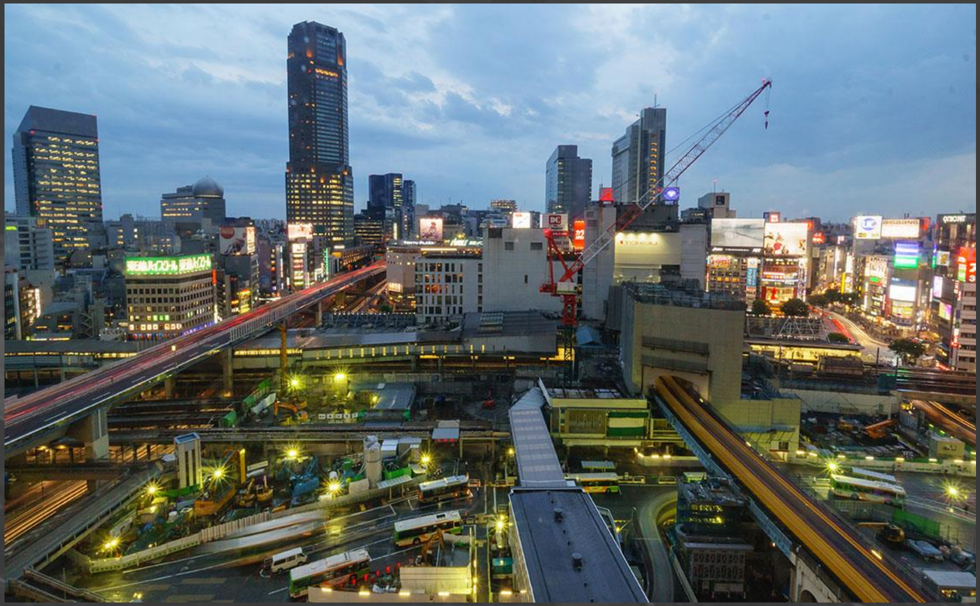
Urban Infrastructures and Their Importance

Urban Infrastructures:

- Essential systems and services necessary for the efficient functioning of a city.
- Includes transportation networks, energy, water and sewage systems, communications, and digital infrastructures.

Importance of Urban Infrastructures:

- Facilitating Daily Life
- Enhancing Quality of Life
- Supporting Local and National Economies
- Resilience Against Crises



Urban Road Network as a Vital Urban Infrastructure

Importance of Urban Road Networks:

- Connectivity and Mobility
- Economic Vitality
- Emergency Response and Safety
- Urban Planning and Development
- Quality of Life

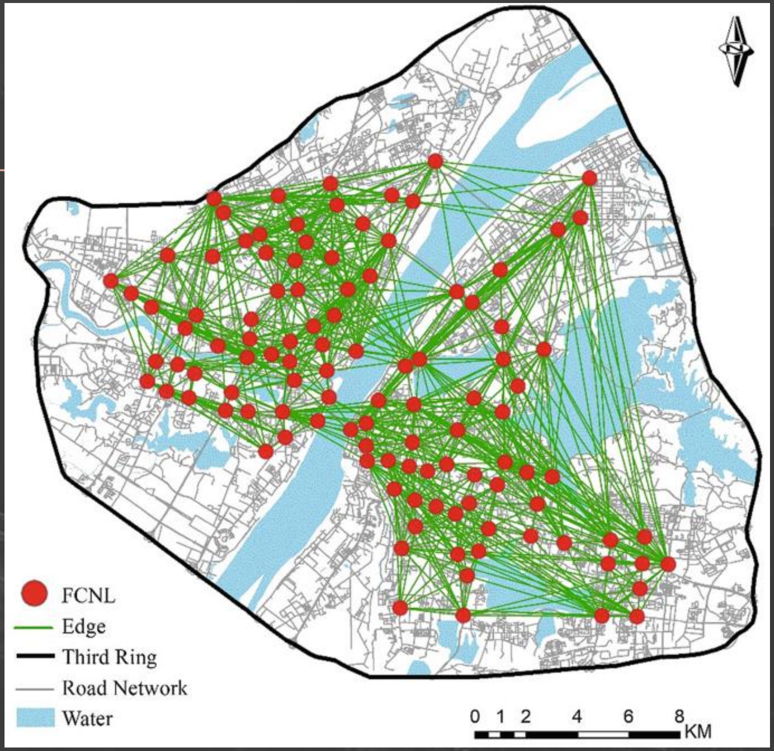
Critical Locations in Urban Road Networks

Characteristics of Critical Locations:

- High Connectivity
- Traffic Volume
- Accessibility
- Emergency Response

Why Identification is Important:

- Resilience Planning
- Efficient Resource Allocation
- Disaster and Crisis Management
- Urban Development



Approaches to Identify Critical Urban Road Network Locations

1. Graph Theory-Based Methods:

- Use structural metrics like node degree, clustering coefficient, and centrality measures.
- ✓ Advantages: Provide a clear understanding of network topology and connectivity.
- o Disadvantages: Labor-intensive and computationally demanding, especially for large, dynamic networks.

2. Link Elimination Analysis:

- Examine the impact of removing specific network links on overall performance.
- ✓ Advantages: Highlights the importance of individual links in maintaining network integrity.
- o Disadvantages: Time-consuming and requires frequent updates with changing traffic patterns.

3. Data-Driven Methods:

- Utilize large databases to analysis traffic flow and congestion impacts.
- ✓ Advantages: Leverage real-time data for accurate and dynamic modeling.
- o Disadvantages: Require extensive datasets and continuous updates, challenging for resource-constrained environments.

Approaches to Identify Critical Urban Road Network Locations

Traditional Approaches:

4. Kernel Density Estimation (KDE):

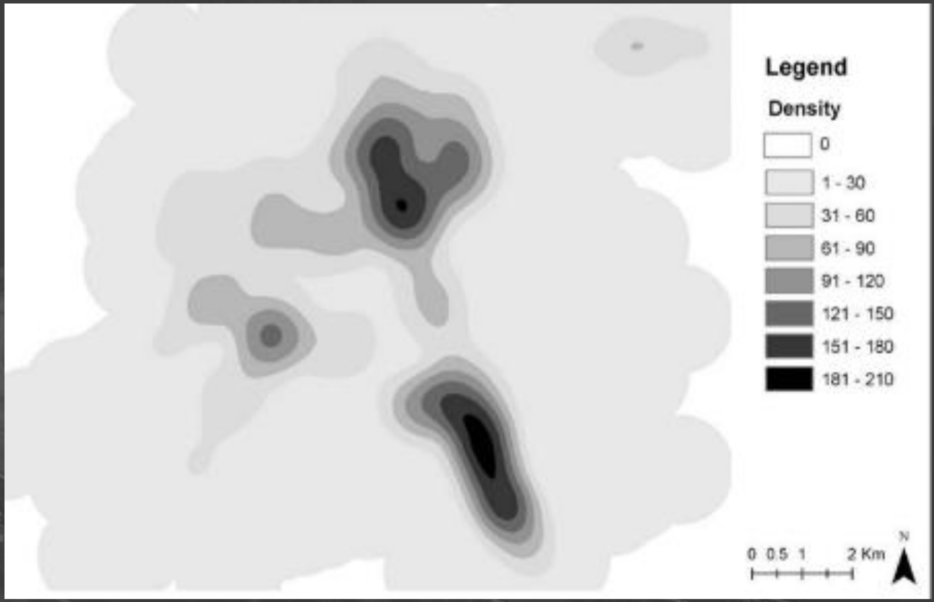
- Uses spatial point pattern analysis to identify critical locations.

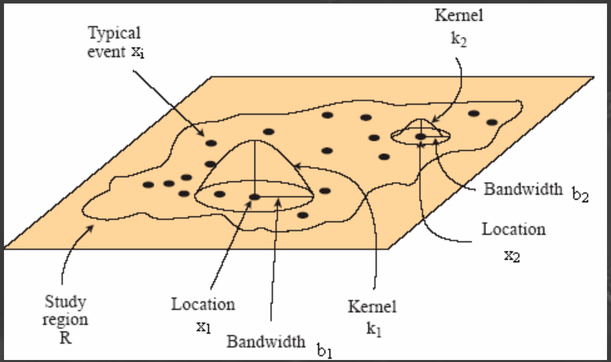
✓ Advantages: Combines structural characteristics with traffic flow data, providing nuanced insights.

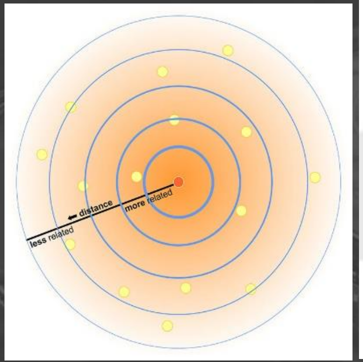
○ Disadvantages: Ordinary KDE may lack alignment with the actual road network, relying on Euclidean distance.

Kernel Density Estimation (KDE) Steps:

- Sample Points Selection
- Estimation Points Selection
- Bandwidth Selection
- Kernel Function

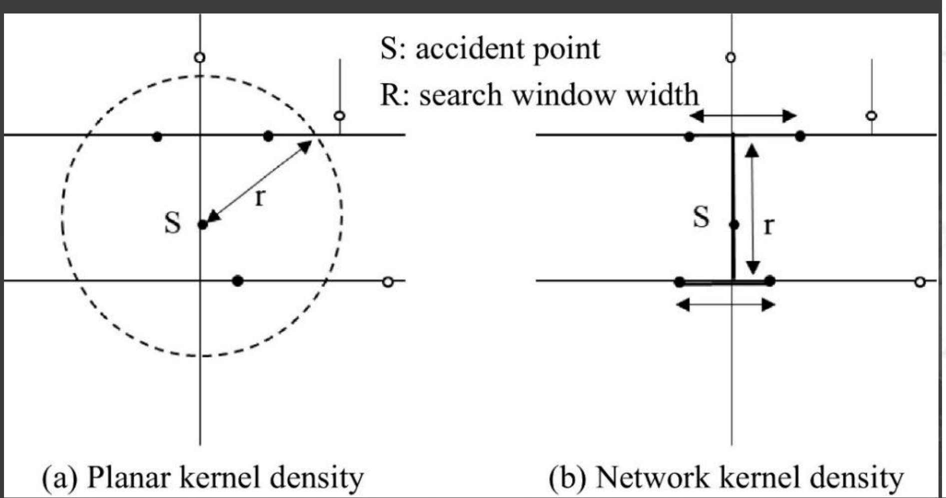






Network Kernel Density Estimation: Bridging the Gap

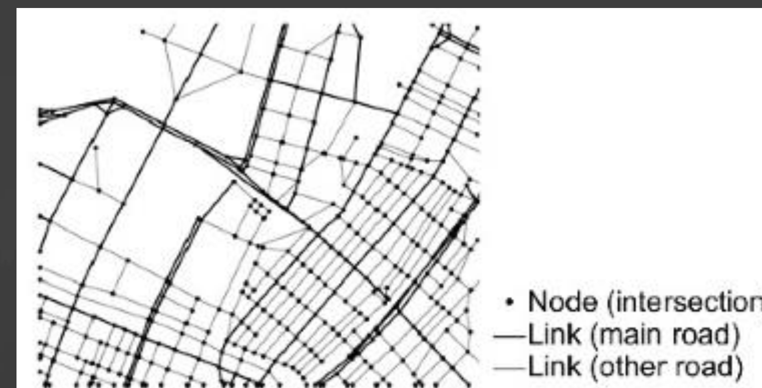
- Network Distance Calculations
- Incorporation of Network Characteristics
- Effective Critical Location Identification



Methodology Overview

This study's methodology comprises several discrete steps:

1. Construction of the Road Network Graph

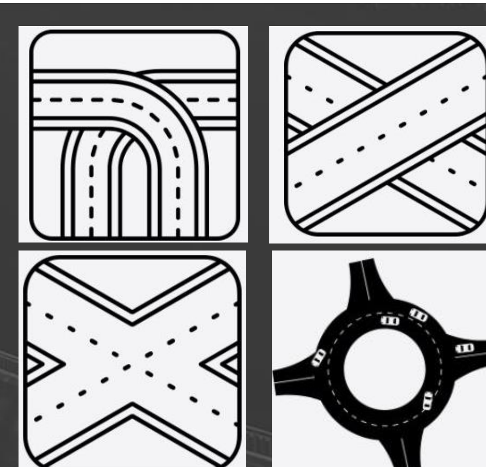


- **Graph Theory Principles:**

The network is represented with road segments as edges and intersections as nodes.

- **Sample Points Selection:**

Samples are obtained from the graph's nodes, focusing on intersections with high public transportation volume.



- **Purpose**:

These sample points function as observed data points to estimate the probability density function.

Methodology Overview

This study's methodology comprises several discrete steps:

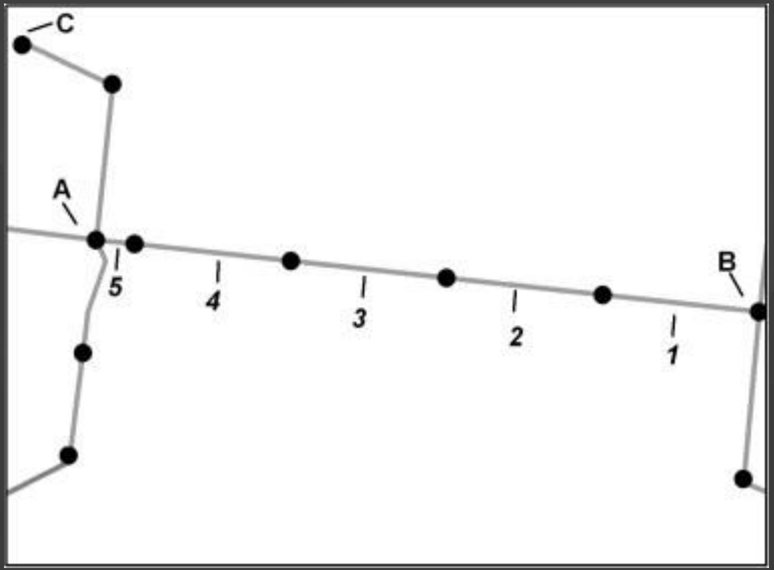
2. Definition of Lixels and Network Construction

- **Segment-Based Graph:**

Road segments between intersections form a segment-based graph.

- **Lixels:**

Each linear lixel unit is a segment divided further, serving as a basic unit in the network.



- **Estimation Points:**

Central points of each lixel act as estimation points, where the probability density function is calculated.

Methodology Overview

This study's methodology comprises several discrete steps:

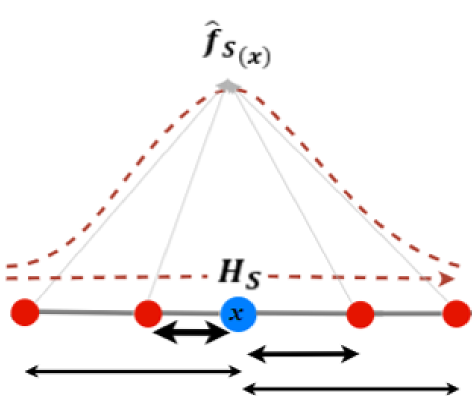
3. Network Kernel Density Estimation (Net-KDE)

- **Density Calculation:**

The spatial density assigned to each estimation point is influenced by nearby sample points on the network.

- **Identifying Critical Locations**

Identify critical locations of the urban transport network based on estimated densities.



$$\hat{f}_{S(x)} = \frac{1}{n}\sum_{i=1}^{n}\sum_{\pi} a(\pi) \times \frac{1}{H_S\sqrt{2\pi}} e^{-0.5\left(\frac{l(\pi)^2}{H_S}\right)}$$

$$a(\pi) = \prod_{v_i \epsilon \pi}\left(\frac{2}{\deg(v_i)} - \delta_i\right)$$

$$H_S = \sigma_0\left(\frac{\left(\frac{\tilde{f}_{S(x)}}{n}\right)^{-0.5}}{\prod_{i=1} {a_i}^{1/n}}\right)$$

Study Area and Data Collection

Study Area

- **Location: Tehran, the capital city of Iran.**
- **Challenges:**
  - **Natural disasters like earthquakes and floods.**
  - **Overcrowded transit system complicating crisis management.**

Data Collection

- **Sources of Data:**
  - Tehran Municipality's Public Bus Transport Data (2020–2021).
  - Road network datasets

Implementation and Results

Sampling Process and Selection of Estimation Points

- **Strategic Selection:**
  - Samples and estimation points chosen to represent Tehran's urban transport system.
  - Intersections prioritized based on characteristics like four-way connections and high-traffic-volume.

- **Focused Representation:**
  - High-traffic intersections chosen to capture important nodes in the urban transport system.
  - Insights into public transportation operations and urban mobility obtained from this strategic selection.

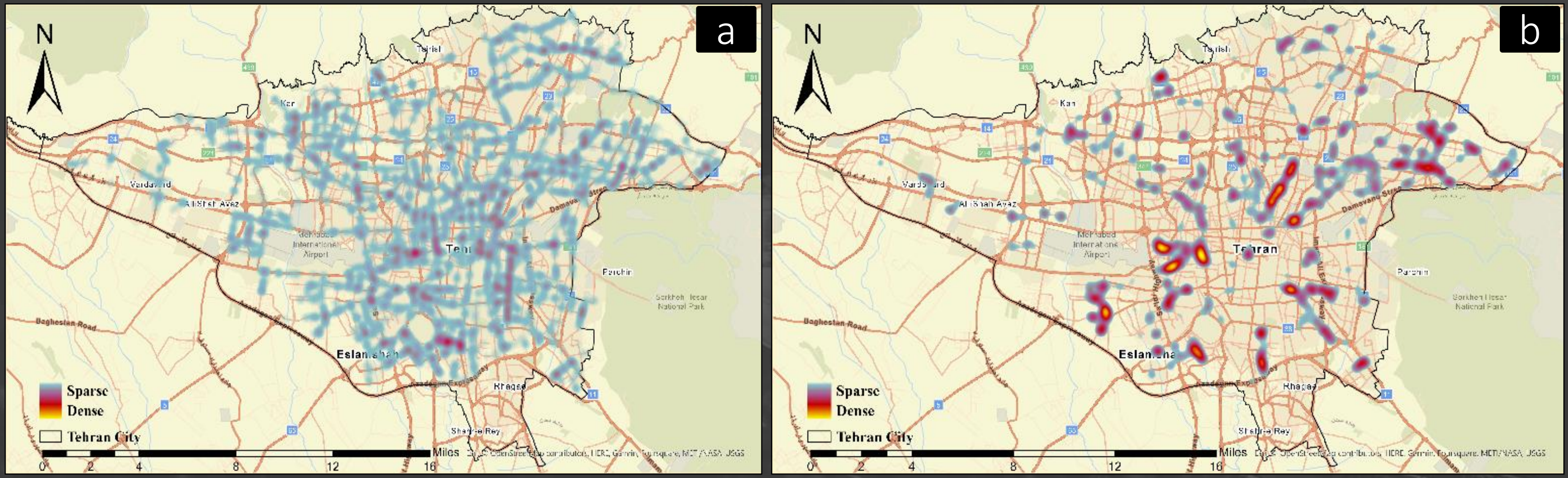


Figure 4. Heat map of Samples Spatial Distribution (a) Random Sampling, (b) Strategic Sampling

Critical Locations - Methodology

- **Objective:**
  - Evaluate the Net-KDE method’s effectiveness with various sampling strategies and bandwidth selections.

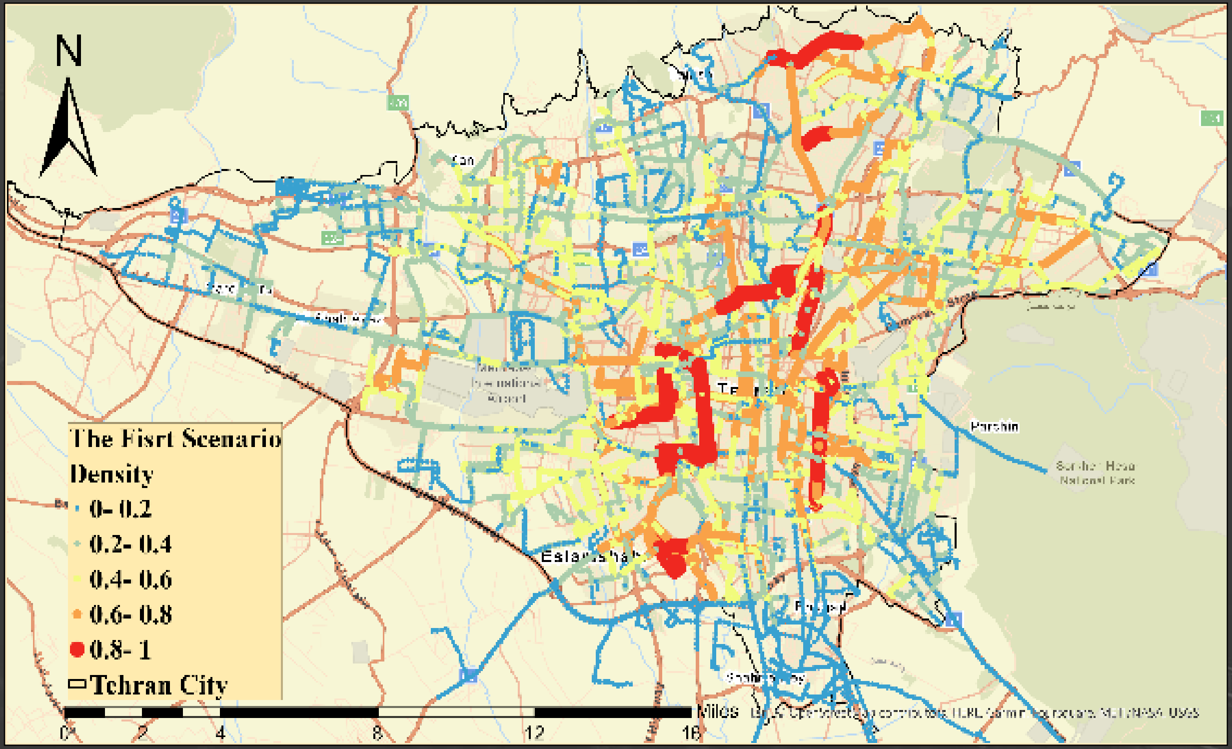
- **Scenarios:**
  - Three scenarios developed to assess effectiveness:
    - Fixed bandwidth with random sampling.
    - Fixed bandwidth with strategic sampling.
    - Adaptive bandwidth with strategic sampling.

Implementation and Results

Scenario 1

- **Approach:**
  - Net-KDE with 5000 randomly selected sample points.
  - Fixed bandwidth determined at 1.3 km.

- **Objective:**
  - Capture local density variations while avoiding overfitting.



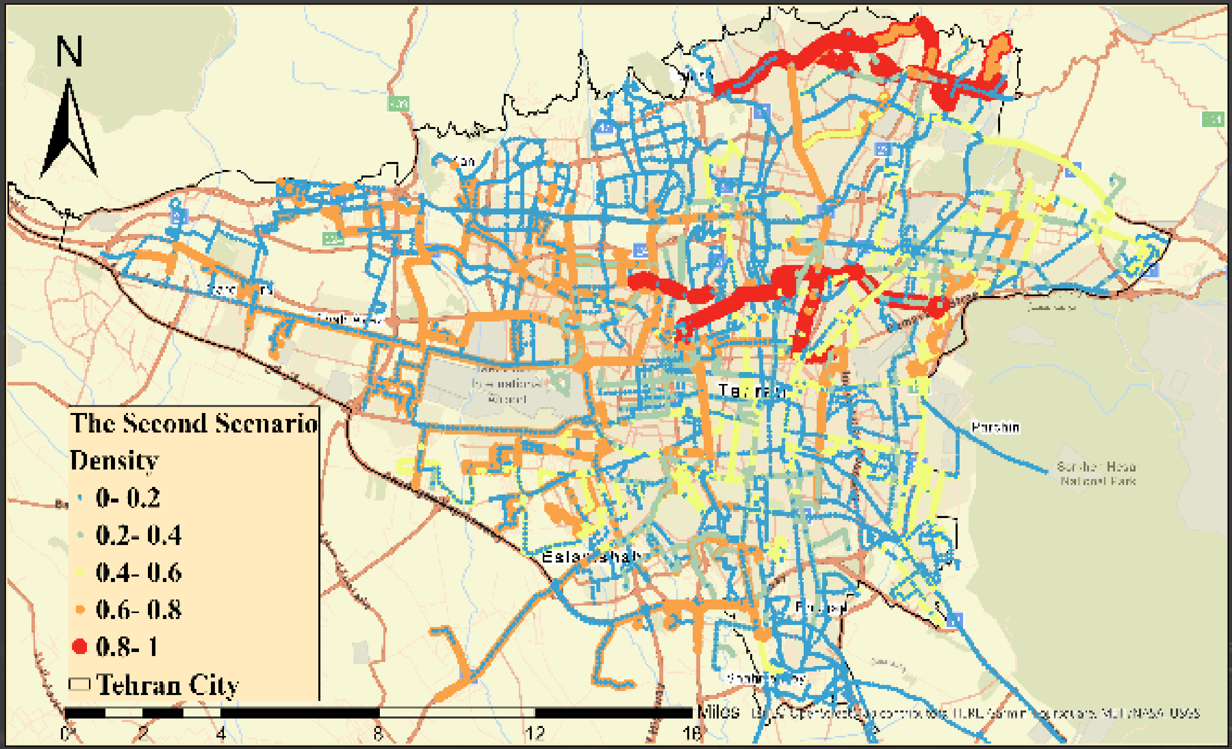
Implementation and Results

Scenario 2

- **Approach:**

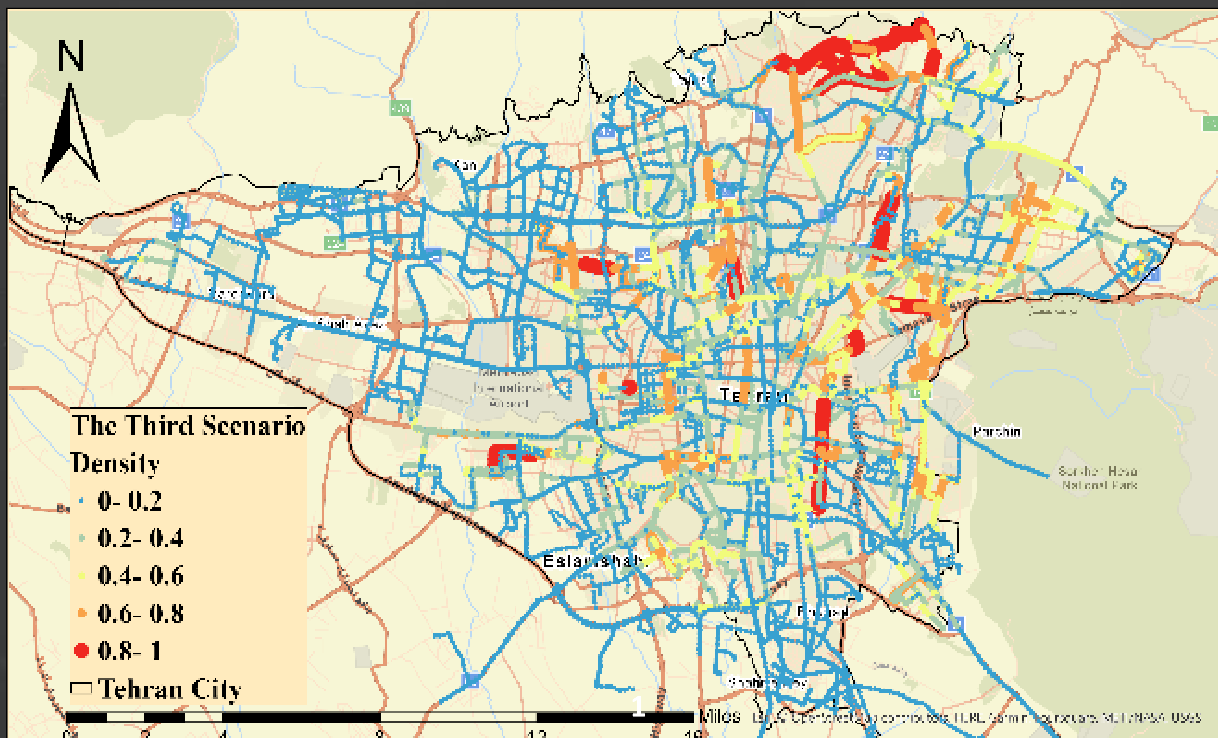
- Net-KDE with 1,411 strategically selected sample points.
- Fixed bandwidth determined at 2.9 km.

- **Objective:**
  - Focus on specific areas of interest within the urban transport network.



Scenario 3

- **Approach:**
  - Net-KDE with strategically selected sample points.
  - Adaptive bandwidth determined, ranging from 0.81 km to 3.6 km.
- **Objective:**
  - Dynamically adjust to spatial characteristics of sample points for nuanced density estimation.

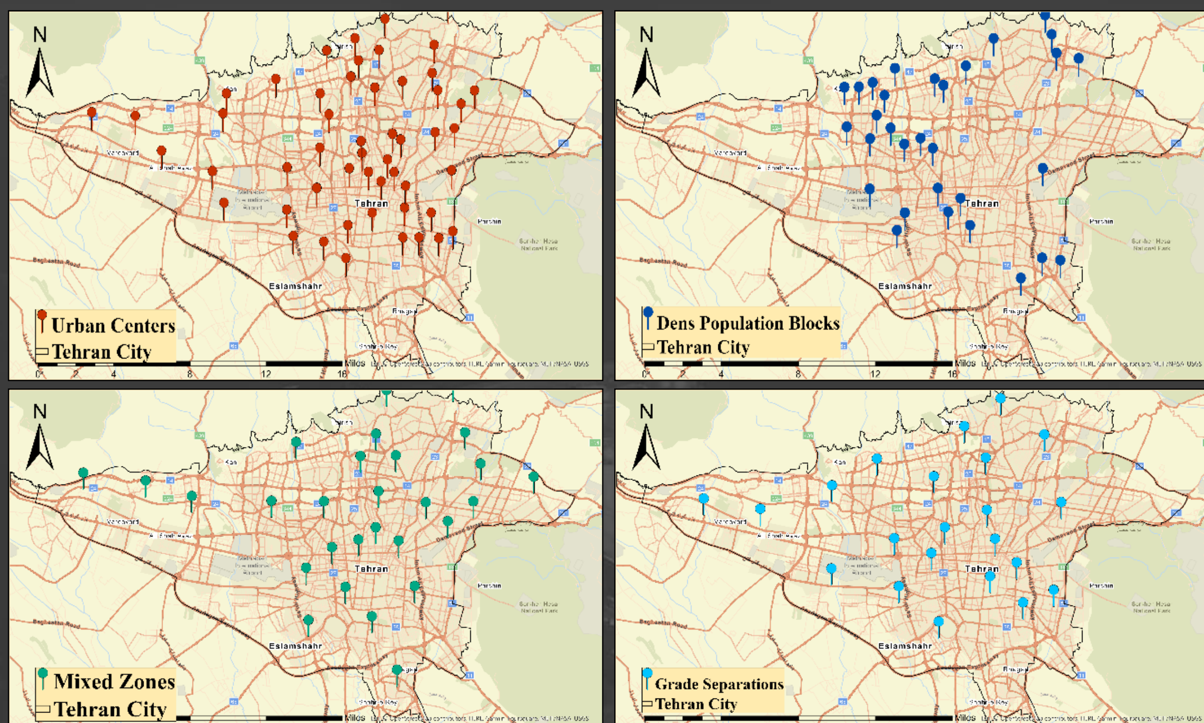


Evaluation

Performance Evaluation: Control Maps

- **Thematic maps of Tehran used as controls, including:**
    - Urban centers
    - Dense population blocks
    - Mixed-use areas
    - Grade separations

Serve as controls for determining critical locations identified by various scenarios.



Evaluation

Spatial Similarity Evaluation:

- Ripley's K-Function: Higher values indicate spatial clustering, lower values indicate dispersion.
- Average Nearest Neighbor (ANN) Ratio: Determines distance between each point and its nearest neighbor.

$$K(r) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} I(\|x_i - x_j\| \leq r)$$

$$ANN\ Ratio = \frac{Observed\ Mean\ Distance}{Expected\ Mean\ Distance}$$

- **Purpose:**
  - Assess similarity between scenarios and control maps by examining spatial distribution.

Evaluation

Spatial Similarity Evaluation:

Table 1. The difference between the Ripley's K-Function and ANN indicators for each control map and scenario map

	Urban Centers		Mixed Zones		Grade Separations		Dens Population Blocks	
	K-Function	*ANN*	*K-Function*	*ANN*	*K-Function*	*ANN*	*K-Function*	*ANN*
Scenario1	2.018	1.7699	3.327	2.9061	2.1505	3.2302	0.2373	1.5298
Scenario2	5.1366	1.9749	6.445	3.111	5.2685	3.4352	2.881	1.2653
Scenario3	1.7466	1.6969	3.055	2.833	1.8785	3.1572	0.5093	0.9874

Evaluation

Spatial Similarity Evaluation:

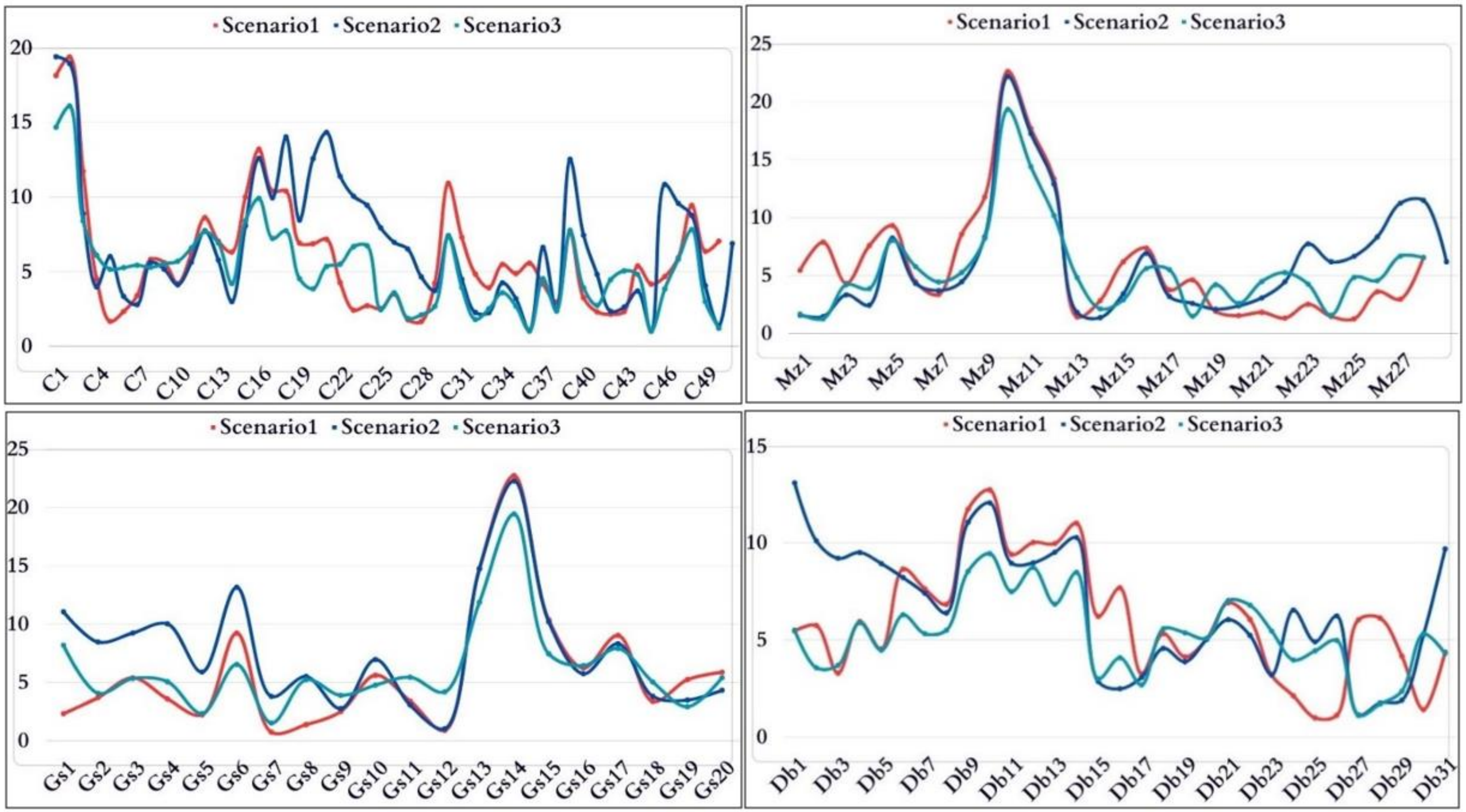


Figure 9. Average distance (Kilometers) of the control points (a) Urban Centers, (b) Mixed zones, (c) Grade separations, (d) Dens population blocks from the K nearest critical locations

Conclusion and Future Directions

- **Practical Applications:**
  - Enhanced Crisis Management
  - Urban Development
  - Traffic Management

- **Future Directions:**
  - Temporal Analysis
  - Additional Parameters
  - More Data Sources
  - Optimization Techniques

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